CIRCULAR JET IN FIELD OF ARCHIMEDES FORCES WITH VARIABLE COEFFICIENT OF THERMAL EXPANSION

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Results of calculations are presented pertaining to an axisymmetric laminar jet of an incompressible fluid and covering the entire region of flow in a field of Archimedes forces. The coefficient of thermal expansion of the fluid is assumed to be a linear function of temperature.

The equations of a boundary layer were used in an earlier study [1] for analytically and numerically solving the problem of an axisymmetric jet of an incompressible fluid heated and vertically upward discharged into an isothermal medium by Archimedes forces of its initial momentum. As an extension of that study [1], there will now be given a numerical solution to this problem for the case of a fluid with a temperature-dependent coefficient of thermal expansion.

In reality the coefficient of thermal expansion is indeed a function of temperature, which appreciably influences the momentum and heat transfer during motion of a fluid [2, 3]. Transformation of the equations of motion and heat transfer in the boundary-layer approximation [4] as well as the method of their integration have already been shown [1]. These equations are

$$\frac{\partial u}{\partial \zeta}=\frac{\Delta T^2}{\eta}\frac{\partial}{\partial \eta}\left(\frac{uy^2}{\eta}\frac{\partial u}{\partial \eta}\right)+\frac{\Delta T}{\eta^2}\left(1-\frac{1}{\mathrm{Pr}}\right)y^2u\frac{\partial \Delta T}{\partial \eta}\frac{\partial u}{\partial \eta}+\frac{\mathrm{Gr}}{\mathrm{Re}}(1+\alpha\Delta T)\frac{\Delta T}{u}, \tag{1}$$

$$\frac{\partial \Delta T}{\partial \zeta}=\frac{\Delta T^2}{\mathrm{Pr}\,\eta}\frac{\partial}{\partial \eta}\left(\frac{uy^2}{\eta}\frac{\partial \Delta T}{\partial \eta}\right),$$

$$V=-\frac{u}{y}\int_0^{\eta}\frac{1}{u^2}\frac{\partial}{\partial \eta}\left(\frac{\Delta T y^2 u}{\eta}\frac{\partial u}{\partial \eta}\right)d\eta-\frac{\mathrm{Gr}}{\mathrm{Re}}(1+\alpha\Delta T)\,u\int_0^{\eta}\frac{\eta d\eta}{u^3}$$

with initial and boundary conditions for the zone of integration

$$\begin{array}{ll} \zeta=0 & \zeta\geqslant 0 \\ \Delta T=u=1,\quad 0\leqslant\eta<1, & \dfrac{\partial \Delta T}{\partial \eta}=\dfrac{\partial u}{\partial \eta}=V=0,\quad \eta=0, \\ \Delta T=u=0,\quad \eta=1. & u=\Delta T=0,\quad \eta=1. \end{array} \tag{2}$$

The results of a numerical solution of the system of equations (1), (2) are shown in Fig. 1 and are compared in Fig. 2 with the results for such an axisymmetric jet of a fluid whose coefficient of thermal expansion β is constant, the numerical solution for the latter case agreeing closely with the solution obtained by the method of local self-adjointness [1].

When the coefficient of thermal expansion changes with the temperature, then the jet flow is non-self-adjoint throughout the entire flow region. At high values of parameter α, nevertheless, the Archimedes force is approximately equal to $\beta_m \alpha g \Delta T^2$ and application of the method of local self-adjointness will yield relations for the scale magnitudes of jet parameters which characterize the motion of the fluid:

$$I=\sqrt{1+\frac{8}{3}\frac{\mathrm{Gr}\,x}{\mathrm{Re}}},\quad u_m=\frac{3}{8x}\sqrt{1+\frac{8}{3}\frac{\mathrm{Gr}\,x}{\mathrm{Re}}},$$
$$\Delta T_m=\frac{3}{8x},\quad \delta=\frac{\sqrt{\frac{8}{3}x}}{\left(1+\frac{8}{3}\frac{\mathrm{Gr}\,x}{\mathrm{Re}}\right)^{1/4}}. \tag{3}$$

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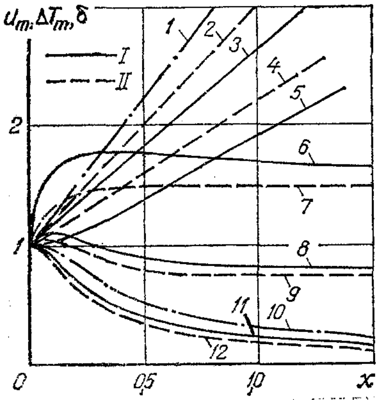

Fig. 1

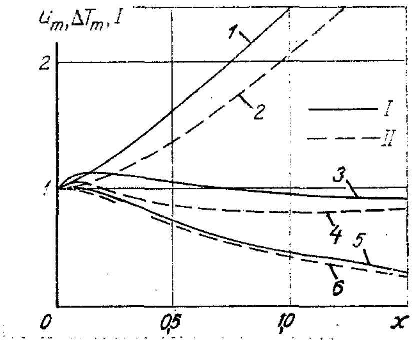

Fig. 2

Fig. 1. Velocity, temperature, and width of jet as functions of x-coordinate: Pr = 0.72, I) $\alpha = 1$, II) $\alpha = 0$, curves 1-5) $\delta = \delta(x)$; 1) Gr/Re = 0; 2, 3) 1; 4, 5) 5; curves 6-10) $u_m = u_m(x)$; 6, 7) Gr/Re = 5; 8, 9) 1; 10) 0; curves 11-12) $\Delta T_m = \Delta T_m(x)$; 11) Gr/Re = 1.5; 12) 0, 1, 5.

Fig. 2. Comparison of scale magnitude of jet parameters with β = var and β = const respectively. Pr = 2; I) $\alpha = 1$; II) $\alpha = 0$; Gr/Re = 1; 1, 2) I; 3, 4) u_m; 5,6) ΔT_m.

Far away from the jet discharge point these scale magnitudes vary as functions of the x-coordinate according to the respective relations

$$I \propto \sqrt{x}, \quad u_m \propto 1/\sqrt{x}, \quad \Delta T_m \propto 1/x, \quad \delta \propto x^{3/4}. \tag{4}$$

When the coefficient of thermal expansion is constant [1], then $u_m \sim$ const. It is quite evident that the mode of u_m variation is somewhere intermediate between u_m = const and $u_m \propto 1/\sqrt{x}$ (curve 3 in Fig. 2).

In conclusion, we will point out a few physical characteristics of a jet with a variable coefficient of thermal expansion. The propagation of such a jet during mixed convection is influenced simultaneously by Archimedes forces, inertia forces, and viscous forces.

The results of calculations reveal that near the point of jet discharge the influence of hydrodynamic forces is quite strong, while the Archimedes forces become the principal driving forces as the distance from the nozzle increases. It should be noted that the given method of calculation is applicable to a jet with mixed convection. The characteristics of a convective jet flow induced by Archimedes forces cannot be obtained by this method directly, because the initial excess heat content is in this case zero so that the region of integration degenerates into a straight line. As the parameter Gr/Re tends to infinity, however, the solution to the system of Eqs. (1) for initial and boundary conditions (2) will describe the behavior of a convective jet.

The temperature dependence of the coefficient of thermal expansion plays also an important role from the standpoint of control of jet flow processes. The variation of the scale magnitudes as functions (4) of distance from the jet discharge point indicates that one can vary the qualitative flow pattern by varying the parameter α. This is particularly noticeable when α becomes negative (Eqs. (1)). Within jet cross sections the Archimedes forces reverse direction (change sign from negative near the jet axis to positive near the jet periphery) so that the flow pattern becomes quite intricate. The temperature dependence of the coefficient of thermal expansion β boosts the effect of fluid acceleration near the discharge point, this effect being less pronounced in the case of β = const. At the same time, it slows down the jet expansion. It must be pointed out that the variation of u_m as function of distance from the jet discharge point remains non-self-adjoint throughout the entire flow region. Far away from that point u_m has both upper and lower limits (self-adjoint asymptotes), u_m = const and $u_m \propto 1/\sqrt{x}$, respectively. We thus conclude that the temperature dependence of the coefficient of thermal expansion appreciably influences the propagation of a jet in a field of Archimedes forces.

NOTATION

ν, kinematic viscosity of the fluid; a, thermal diffusivity of the fluid; d, nozzle diameter; u_0, fluid velocity at the nozzle throat; T^*, jet temperature; T_0^*, T_m^*, and T_∞^*, temperatures in the nozzle throat, at the jet axis, and of the ambient medium respectively; β, coefficient of thermal expansion of the jet; β_m, coefficient of thermal expansion at the temperature $T_m^* = {}^1/_2(T_0^* + T_\infty^*)$; $\alpha = \alpha^*(T_0^* - T_\infty^*)$; $\alpha^* = \frac{1}{\beta_m}\left(\frac{\partial\beta}{\partial T}\right)_P$, a dimensionless parameter characterizing the dependence of β on T^*; $Re = u_0 d/\nu$, Reynolds number; $Pr = \nu/a$, Prandtl number; $Gr = \beta_m g d^3(T_0^* - T_\infty^*)/\nu^2$, Grashof number; $x = x^*/Red$ and $y = y^*/d$, dimensionless coordinates, longitudinal and transverse, respectively; $u = u^*/u_0$, $V = (V^*/u_0)$ Re, dimensionless velocity, longitudinal and transverse, respectively; u_m, dimensionless velocity at the jet axis; $\Delta T^* = T_0^* - T_\infty^*$, excess temperature in the nozzle throat; $\Delta T = (T^* - T_\infty^*)/(T_0^* - T_\infty^*)$, dimensionless excess temperature of the jet; and δ, conventional jet width (distance from jet axis to point where $u = {}^1/_2 u_m$, $\delta = \delta^*/d$ being the dimensionless jet width).

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CONDITIONS FOR MIXING TRANSVERSE CO_2 JETS WITH A SUPERSONIC NITROGEN STREAM IN A NOZZLE

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A flow visualization experiment has been performed in a planar supersonic nozzle with strong transverse jet blowing of CO_2. The authors have established the existence of two regimes for interaction of the jets with the main stream and with themselves. They have determined the dimensions of the zone where this interaction is strongest.

Jet blowing of gas into supersonic flows is widely used in contemporary facilities. The interaction of streams with a single jet has been investigated by a number of authors, e.g., [1-4]. In all these studies, apart from [4], they considered flows in channels of constant or slightly varying cross section with a small relative flow rate of blown gas. In [4], which studied flow in a nozzle, the flow rate of blown gas did not exceed 4% of the main stream rate. The flow structure in the conditions indicated varied appreciably only in the vicinity of the blowing location and was studied well. For the mixing nozzles with large relative flow rate of blown gas used in [5-8], it is of interest to study the interaction of a number of transverse jets among each other and with the main supersonic stream. The principal difference between this kind of flow and flows with transverse blowing of a single jet is that the perturbations brought in by the blown gas are not local, but change the structure of the entire flow. Apart from an investigation of the process of starting a nozzle with blowing, described in [6], the literature has no information on the structure of such flows.

In the present work flow visualization has been used to obtain pictures of the structure of flows and evaluate the dimensions of the zone of strong interaction of flows with transverse blowing of a number of CO_2 jets into a nozzle in an expanding flow of a mixture of a number of gases based on nitrogen, over a wide range of blown gas flow rate.

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